**Baseball Elimination**

Until 1993, two divisions in each league with only 4 playoff teams.

In 1995, a wildcard was added.

**AL East standings, August 30, 1996.**

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<table>
<thead>
<tr>
<th>Team</th>
<th>Wins</th>
<th>Losses</th>
<th>To Play</th>
<th>Against each other</th>
</tr>
</thead>
<tbody>
<tr>
<td>NY</td>
<td>75</td>
<td>59</td>
<td>28</td>
<td>- 5 7 4 3</td>
</tr>
<tr>
<td>Baltimore</td>
<td>71</td>
<td>63</td>
<td>28</td>
<td>5 - 2 4 4</td>
</tr>
<tr>
<td>Boston</td>
<td>69</td>
<td>65</td>
<td>28</td>
<td>7 2 - 4 0</td>
</tr>
<tr>
<td>Toronto</td>
<td>63</td>
<td>71</td>
<td>28</td>
<td>4 4 4 - 0</td>
</tr>
<tr>
<td>Detroit</td>
<td></td>
<td></td>
<td></td>
<td>3 4 0 0 -</td>
</tr>
</tbody>
</table>

NY Ba Bo TD
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All that most sports writers know is: \( w_i + r_i < w_j \) 

Team \( i \) is eliminated if \( w_i + r_i < w_j \)

If Detroit \( w_5 = 46, l_5 = 88, 46 + 28 = 74 < 75 \).

Sufficient, but not necessary!

What if \( w_5 = 47 \)? Still eliminated because \( w_5 + r_5 = 75 \) and either NY or Baltimore will win 76 games since they play each other 5 times!
Is Detroit Eliminated?

Eliminated if \( W_5 = 46 \) or \( 47 \). \( r_5 = 28 \)

What about \( W_5 = 48 \) (\( L_5 = 86 \))? \( \forall \) all cases

More difficult analysis but can show elimination.

Use max-flow!

Flow network to determine if team 5 is eliminated.

Capacities are number of games team i can win and not have more wins than team 5

Intuition: Assume team 5 wins all remaining games. Divvy up remaining games so all teams have \( \leq W_5 + r_5 \) wins.

Capacities are games to be played between i and j.
CLAIM

Theorem: Team 5 is eliminated iff max-flow does not saturate all edges leaving source. (i.e., max flow value < 26).
(Saturation corresponds to playing all the remaining games.)

Argument: If you can't play all the remaining games without exceeding the capacities of \( i \rightarrow T \) edges, team 5 is eliminated.

Look at min-cut of our example

\[(S, T)\] above is min-cut. \( c(S, T) = 4+4+4+1+5+7 = 25 \Rightarrow \text{elimination.} \]
Look at subset of teams, namely, 1, 2, 8, 3.

\[ W_1 + W_2 + W_3 = 75 + 71 + 69 = 215 \text{ wins} \]

Teams play each other 5 + 7 + 2 = 14 times.

Total number of guaranteed wins

= 215 + 14 = 229

Observe \[ \frac{229}{3} = 76.33 \]

At least one of those teams will win 77 games.
(Detroit can win 48 + 28 = 76)

Max-flow min-cut finds this subset if it exists.