18.510 Introduction to Mathematical Logic and Set Theory

Course Information
Meetings: TR 2:30–4:00 in 4–153
Instructor: Henry Cohn
Office: 2–341B [I’m normally at Microsoft Research, so you won’t generally find me in Building 2 except for office hours and other scheduled appointments.]
Office Hours: Tuesday 4:00–5:00, Thursday 1:30–2:30, and by appointment
E-mail: cohn@microsoft.com

Grading: ten homework assignments 50%, two exams 25% each. Specifically, your eight best homework assignments will count for 6% each, the second lowest will count for 2%, and the lowest will be dropped entirely. If you end up on the border between two grades, I’ll give you the higher grade if I feel your performance has improved during the course.

Due dates: homework is due at the beginning of class on the specified date. If you aren’t in class that day, please send it with someone else or send a scan by e-mail (there are scanners in the library).

Homework turned in later on the due date will be penalized 10%, and no homework will be accepted after the due date, except because of medical or family emergencies or religious obligations. The decreased weight for the lowest homework scores is meant to accommodate other issues (being busy with classes, athletics, or job searches, feeling unproductive due to an ordinary cold, etc.).

Exams: Thursday, October 19 and Thursday, December 7, during class.
Both will be closed-book exams. The second exam will focus on the second half of the course, rather than being a comprehensive final exam. I’ll give more details and sample exams as the exam dates get closer.

Prerequisites: in principle, none. Everything we do will be built up from scratch and won’t depend on knowledge from previous courses. In practice, 18.510 requires a certain level of mathematical maturity and comfort with abstraction and proofs. If you have never taken a proof-based mathematics class before, you
may have trouble with this course. Although it starts from scratch, it is not an elementary or low-level course.

Text: none. I’ll distribute lecture notes, and I’m happy to suggest relevant books. For the material on set theory towards the start of the course, here are some references:

- *Naive set theory* by Paul R. Halmos
- *Basic set theory* by A. Shen and N. K. Vereshchagin
- *Set theory, logic and their limitations* by Moshé Machover

Attendance policy: you are adults, so I won’t take attendance or grade based on it. However, because we won’t be following a textbook, I strongly recommend attending class regularly. If you miss class, you should get notes from someone.

Collaboration policy: working with classmates is fine (indeed, encouraged), but you should write up everything on your own, and of course you should understand everything you turn in. When you work with other students, you should be collaborating to solve problems together, rather than getting out of work by sharing solutions. It’s OK to use books and web sites, and it’s possible that you will occasionally run across a solution to a homework problem, but you should not deliberately search for solutions, and if you read a solution in the literature you should explain it in your own words (rather than copying it verbatim). You should not post homework questions to Q&A web sites. If you collaborate with someone on a problem or learn a solution from some other source, you should acknowledge this on your problem set.

If you would like to work with other students but lack a group of friends taking the class, please let me know so I can put you in touch with each other.

Course outline: In the first half of the course, we’ll study set theory, beginning with the ZFC axioms. At first, we’ll mainly analyze familiar things rigorously (what is a finite set anyway, etc.), but then we’ll move on to studying ordinals and cardinals. We’ll develop a careful and powerful theory of transfinite numbers, and we’ll look at some equivalent forms of the axiom of choice. Finally, we’ll study cardinal arithmetic and cofinality. At this point, we’ll be able to state and prove some really beautiful and non-obvious theorems about infinite cardinals. The first exam will cover this half of the course.

In the second half, we’ll develop the theory of first-order logic. We’ll cover some proof theory (what exactly is a proof) and model theory (what can one say about the mathematical structures satisfying a set of axioms). In particular, we’ll study Henkin’s proof of the completeness theorem, as well as a proof of the compactness theorem using ultrafilters. Once we have developed this machinery, we’ll take a look at Gödel’s incompleteness theorems. The second exam will cover this half of the course.

There are many connections between the two halves of the course. For example, we’ll use some set theory in our development of first-order logic, while first-order logic will inform how the axioms of set theory are stated. However, the most dramatic connection will come at the end of course. We’ll combine the incompleteness theorems with the concept of large cardinals (truly vast transfinite numbers) to see how mathematical logic constrains set theory.