Unsupervised discovery of neural sequences in large-scale recordings

github.com/Feelab/seqNMF

BioRxiv preprint
Neurons form sequences

Songbird HVC (premotor cortex), Okubo et al. 2015
Neurons form sequences

Rat hippocampus, Pastalkova et al. 2008

Rat posterior parietal cortex, Harvey et al. 2012

Macaque medial frontal cortex, Wang et al. 2018

Simulated recurrent network, Rajan et al. 2017
Behavioral sequences are also relevant to neuroscience

Kristin Branson lab,
see JAABA behavioral analysis tool
CNMF_E cell extraction (PC Zhou, Paninski lab, with Shijie Gu)
Neurons
(re-sorted)
Non-negative matrix factorization (NMF)

(like PCA/SVD, but factors must be positive)
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(like PCA/SVD, but factors must be positive)
Non-negative matrix factorization (NMF)

\[
W \leftarrow W \times \frac{XH^T}{WHH^T}
\]

\[
H \leftarrow H \times \frac{W^TX}{W^TWH}
\]

Lee and Seung 1999
Convolutional NMF

\((\tilde{W}, \tilde{H}) = \arg\min_{W,H} \left( \|X - W \otimes H\|_F^2 \right)\)

Convolutional NMF

In practice ... redundant factors

\[(\tilde{W}, \tilde{H}) = \arg \min_{W,H} \left( \|X - W \otimes H\|_F^2 \right)\]
In practice ... redundant factors

\[
(W_1, W_2) = \arg \min_{W,H} \left( \|X - W \otimes H\|_F^2 \right)
\]
In practice ... redundant factors

\[ (\tilde{W}, \tilde{H}) = \arg\min_{W,H} \left( \|X - W \otimes H\|_F^2 + \lambda \mathcal{R} \right) \]

Penalty for correlations
In practice ... redundant factors

\[
(W_1, W_2) = \arg \min_{W, H} \left( \|X - W \odot H\|_F^2 + \lambda \mathcal{R} \right)
\]

\[
\mathcal{R} = \|HH^T\|
\]
In practice ... redundant factors

\( (\tilde{W}, \tilde{H}) = \arg\min_{W, H} \left( \|X - W \odot H\|_F^2 + \lambda \mathcal{R} \right) \)

\[ \mathcal{R} = \|HSH^\top\| \]

Penalty for correlations
In practice ... redundant factors

\[
(W_1, W_2) = \arg \min_{w, h} (||X - W \otimes H||_F^2 + \lambda \mathcal{R})
\]

\[
\mathcal{R} = ||HSH^\top||
\]
In practice ... redundant factors

\[(\tilde{W}, \tilde{H}) = \arg \min_{W,H} (\|X - W \odot H\|_F^2 + \lambda \mathcal{R}) \]

\[\mathcal{R} = \|W^\top \odot XSH^\top\|\]
Simple multiplicative update rules

\[
\begin{align*}
W_{\downarrow} &\leftarrow W_{\downarrow} \times \frac{X H^T}{\tilde{X} H^T + \lambda XSH^T (1 - I)} \\
H &\leftarrow H \times \frac{W \otimes X}{W^T \tilde{X} + \lambda (1 - I)(W \otimes XS)}
\end{align*}
\]

\[
(\tilde{W}, \tilde{H}) = \arg \min_{W,H} \left( \|X - W \otimes H\|_F^2 + \lambda \mathcal{R} \right)
\]

\[
\mathcal{R} = \|W^T \otimes XSH^T\|
\]

Mackevicius and Bahle, et al., 2018
Testing seqNMF on simulated sequences

Mackevicius and Bahle, et al., 2018
Testing seqNMF on simulated sequences

Mackevicius and Bahle, et al., 2018
SeqNMF factorizations are highly consistent

Mackevicius and Bahle, et al., 2018
Testing significance of each factor on held-out data

Many moments of high overlap, compared to null

Mackevicius and Bahle, et al., 2018
SeqNMF discovers the correct number of sequences

Mackevicius and Bahle, et al., 2018
SeqNMF is robust to noise

Mackevicius and Bahle, et al., 2018
Mackevicius and Bahle, et al., 2018
Can SeqNMF discover sequences in small datasets?

Mackevicius and Bahle, et al., in preparation
Can SeqNMF discover sequences in small datasets?

Mackevicius and Bahle, et al., in preparation
Method to choose lambda

\[ (\tilde{W}, \tilde{H}) = \arg \min_{W,H} \left( \|X - W \otimes H\|_F^2 + \lambda R \right) \]

Mackevicius and Bahle, et al., 2018
Method to choose lambda

Mackevicius and Bahle, et al., 2018
Testing SeqNMF on hippocampal sequences

Pastalkova et al., 2008
Testing SeqNMF on hippocampal sequences

Mackevicius and Bahle, et al., 2018
Testing SeqNMF on hippocampal sequences

Mackevicius and Bahle, et al., 2018
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Mackevicius and Bahle, et al., 2018
Testing SeqNMF on hippocampal sequences

Mackevicius and Bahle, et al., 2018
Testing SeqNMF on songbird HVC

A

B

C

Mackevicius and Bahle, et al., 2018
Testing SeqNMF on songbird HVC

Mackevicius and Bahle, et al., 2018
SeqNMF is broadly applicable to many high-dimensional datasets

Hand-labeled song syllables, Okubo 2015
SeqNMF is broadly applicable to many high-dimensional datasets
Agenda

• We hope you now have an idea of whether seqNMF might work on your data
• Extra Tools
  – Cross-validation procedure to estimate the number of sequences
  – Choosing between “parts-based” vs “events-based” factorizations
  – Denoising with convolutional NMF
  – Exercises/link to code
Cross-validation procedure for NMF

See http://alexhwilliams.info/itsneuronalblog/2018/02/26/crossval/
Cross-validation procedure for NMF

Minimize error only in non-masked elements

\[
\arg\min_{W,H} \| M \odot (W H - X) \|_2^2
\]

See http://alexhwilliams.info/itsneuronalblog/2018/02/26/crossval/
Cross-validation procedure for NMF

\[
\arg \min_{W,H} ||M \circ (WH - X)||_2^2
\]

See http://alexhwilliams.info/itsneuronalblog/2018/02/26/crossval/
Cross-validation procedure for convolutional NMF

\[ \arg\min_{W,H} \| M \odot (W \otimes H - X) \|^2_2 \]

\[ \| \sim M \odot (WH - X) \|^2_2 \]
Generalization as a function of model components

Mackevicius and Bahle, et al., in preparation
Songbird HVC

Rat Hippocampus

Mackevicius and Bahle, et al., in preparation
Generalization as a function of penalty

Test Error
Difference
Train Error

Mackevicius and Bahle, et al., in preparation
Choosing between “parts-based” vs “events-based” factorizations

Mackevicius and Bahle, et al., 2018
Choosing between “parts-based” vs “events-based” factorizations

Mackevicius and Bahle, et al., 2018
Choosing between “parts-based” vs “events-based” factorizations

Events-based

Mackevicius and Bahle, et al., 2018
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Mackevicius and Bahle, et al., 2018
Denoising with convolutional NMF

Denoising with PCA

- Original images
- Very noised images
- Denoised images
Exercises

• Download and try the demo code: github.com/Feelab/seqNMF
  – Fit seqNMF
  – Choose lambda
  – Test significance
  – Switch between parts-based and events-based factorizations
  – Try seqNMF on a new dataset